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(k, 1)-Mean Labeling of Some Snake Related Graphs

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Abstract — Mean labeling of graphs was introduced and discussed by Somasundaram and Ponraj. k-mean, k-even mean and (k, d)-even mean labeling are introduced and discussed by Gayathri and Gopi. We have introduced (k, d)-mean labeling and in this paper, we have obtained (k, 1)-mean labeling for some snake related graphs.

Keywords — (k, d)-mean labeling ((k, d)-ML), (k, d)-mean graph ((k, d)-MG).

AMS Subject Classification — 05C78

I. INTRODUCTION

The graphs considered here will be finite undirected and simple. The symbols $V(G)$ and $E(G)$ will denote the vertex set and edge set of a graph G. Let p and q denote the number of vertices and edges of G. For standard terminology and notations we follow Harary [8].

S. Somasundaram and R. Ponraj introduced the concept of mean labeling of graphs in [10].

Gayathri and Gopi [7] extended this concept to k-mean, k-even mean and (k, d)-even mean graphs. In [2] we have introduced (k, d)-mean labeling and obtained results for some family of trees. In [3, 4, 6] we found (k, d)-mean labeling for some special graphs, some disconnected graphs and path related graphs. Some new families of (k, 1)-mean graphs are obtained in [5]. In this paper, we have obtained (k, 1)-mean labeling for some snake related graphs. Throughout this paper, k and d denote any positive integer greater than or equal to 1. For brevity, we use (k, d)-ML for (k, d)-mean labeling and (k, d)-MG for (k, d)-mean graph.

II. MAIN RESULTS

Definition 2.1

A (p, q) graph G is said to have a **(k, d)-mean labeling** if there exists an injective function f from the vertices of G to $\{0, 1, 2, ..., k + (q-1)d\}$ such that the induced map f^* defined on E by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a bijection from E to $\{k, k+d, k+2d, ..., k+(q-1)d\}$.

A graph that admits a (k, d)-mean labeling is called a **(k, d)-mean graph**.

Definition 2.2

An alternate triangular snake $A(T_n)$ is obtained from a path $v_1, v_2, ..., v_{2n}$ by joining v_i and v_{i+1} (alternatively) to a new vertex u_i ($i = 1, 2, ..., n$).

Theorem 2.3

The alternate triangular snake $A(T_n)$ is a (k, 1)-mean graph for all k.

Proof

Let $V(A(T_n)) = \{u_i, 1 \le i \le n, v_i, 1 \le i \le 2n\}$ be the vertices and $E(A(T_n)) = \{e_i, 1 \le i \le 2n-1, a_i, 1 \le i \le 2n\}$ be the edges which are denoted as in Figure 2.1.

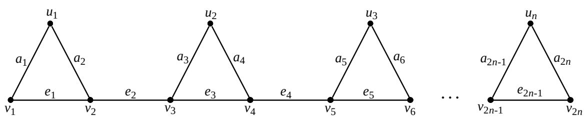

Figure 2.1: Ordinary labeling of $A(T_n)$

First we label the vertices as follows:
Define $f : V \to \{0, 1, 2, ..., k + (q-1)\}$ by

$$f(u_i) = k + 4i - 3, \quad 1 \le i \le n$$

For $1 \le i \le 2n$,

$$f(v_i) = \begin{cases} k+2i-3, & i \text{ odd} \\ k+2(i-1), & i \text{ even} \end{cases}$$

Then the induced edge labels are:

$$f^*(e_i) = k + 2i - 1, \quad 1 \le i \le 2n-1$$

$$f^*(a_i) = k + 2(i-1), \quad 1 \le i \le 2n$$

The above defined function f provides (k, 1)-mean labeling of the graph.

So, the alternate triangular snake $A(T_n)$ is a (k, 1)-mean graph for all k.

Illustration 2.4

(6, 1)-mean labeling of $A(T_5)$ and (2, 1)-mean labeling of $A(T_8)$ are shown in Figure 2.2 and Figure 2.3 respectively.

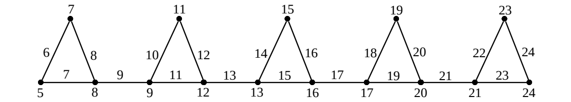

Figure 2.2: (6, 1)-ML of $A(T_5)$

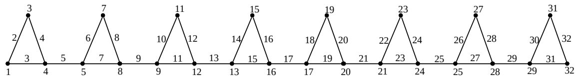

Figure 2.3: (2, 1)-ML of $A(T_8)$

Definition 2.5

An alternate double triangular snake $A(D(T_n))$ is obtained from a path $v_1, v_2, ..., v_{2n}$ by joining v_i and v_{i+1} (alternatively) to new vertices u_i, $u_i^{'}$ ($i = 1, 2, ..., n$).

Theorem 2.6

The alternate double triangular snake $A(D(T_n))$ is a (k, 1)-mean graph for all k.

Proof

Let $V(A(D(T_n))) = \{u_i, 1 \le i \le n, u_i^{'}, 1 \le i \le n, v_i, 1 \le i \le 2n\}$ be the vertices and $E(A(D(T_n))) = \{e_i, 1 \le i \le 2n - 1, a_i, 1 \le i \le 2n, b_i, 1 \le i \le 2n\}$ be the edges which are denoted as in Figure 2.4.

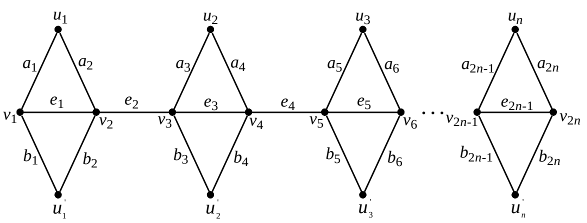

Figure 2.4: Ordinary labeling of $A(D(T_n))$

First we label the vertices as follows:
Define $f : V \to \{0, 1, 2, ..., k + (q - 1)\}$ by

$$f(u_i) = k + 6i - 7, \quad 1 \le i \le n$$
$$f\left(u_i^{'}\right) = k + 6i - 2, \quad 1 \le i \le n$$

For $1 \le i \le 2n$,

$$f(v_i) = \begin{cases} k+3i-2, & i \text{ odd} \\ k+3(i-1), & i \text{ even} \end{cases}$$

Then the induced edge labels are:

$$f^*(e_i) = k + 3i - 1, \quad 1 \le i \le 2n - 1$$

For $1 \le i \le 2n$,

$$f^*(a_i) = \begin{cases} k+3(i-1), & i \text{ odd} \\ k+3i-5, & i \text{ even} \end{cases}$$

$$f^*(b_i) = \begin{cases} k+3i, & i \text{ odd} \\ k+3i+2, & i \text{ even} \end{cases}$$

The above defined function f provides (k, 1)-mean labeling of the graph.

So, the alternate double triangular snake $A(D(T_n))$ is a (k, 1)-mean graph for all k.

Illustration 2.7

(11, 1)-mean labeling of $A(D(T_5))$ and (6, 1)-mean labeling of $A(D(T_6))$ are shown in Figure 2.5 and Figure 2.6 respectively.

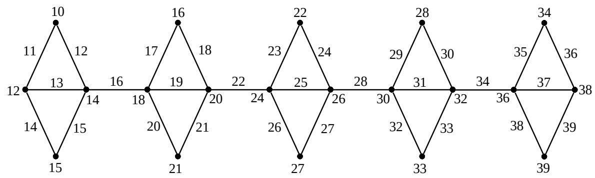

Figure 2.5: (11, 1)-ML of $A(D(T_5))$

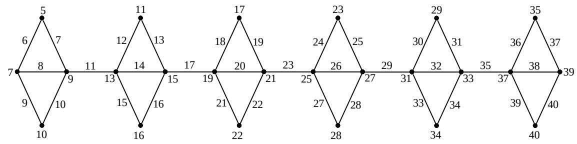

Figure 2.6: (6, 1)-ML of $A(D(T_6))$

Definition 2.8

A quadrilateral snake Q_n is obtained from a path $u_1, u_2, ..., u_{n+1}$ by joining each of the vertices u_i and u_{i+1} ($i = 1, 2, ..., n$) to new vertices v_i and $v_i^{'}$ respectively and then joining v_i and $v_i^{'}$. That is every edge of a path is replaced by a cycle C_4.

Theorem 2.9

The quadrilateral snake Q_n is a (k, 1)-mean graph for all k.

Proof

Let $V(Q_n) = \{u_i, 1 \le i \le n + 1, v_i, 1 \le i \le n, v_i^{'}, 1 \le i \le n\}$ be the vertices and $E(Q_n) = \{a_i, 1 \le i \le n, b_i, 1 \le i \le n, c_i, 1 \le i \le 2n\}$ be the edges which are denoted as in Figure 2.7.

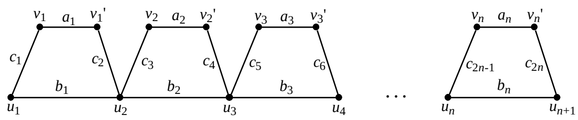

Figure 2.7: Ordinary labeling of Q_n

First we label the vertices as follows:
Define $f : V \to \{0, 1, 2, ..., k + (q - 1)\}$ by

$$f(u_1) = k - 1$$

For $2 \le i \le n + 1$,

$$f(u_i) = \begin{cases} k+4i-5, & i \text{ even} \\ k+4i-6, & i \text{ odd} \end{cases}$$

For $1 \le i \le n$,

$$f(v_i) = \begin{cases} k+4i-3, & i \text{ odd} \\ k+4i-4, & i \text{ even} \end{cases}$$

$$f\left(v_i^{'}\right) = \begin{cases} k+4i-2, & i \text{ odd} \\ k+4i-1, & i \text{ even} \end{cases}$$

Then the induced edge labels are:

$$f^*(a_i) = k + 4i - 2, \qquad 1 \le i \le n$$

For $1 \le i \le 2n$,

$$f^*(c_i) = \begin{cases} k+2(i-1), & i \text{ odd} \\ k+2i-1, & i \text{ even} \end{cases}$$

$$f^*(b_i) = k + 4i - 3, \qquad 1 \le i \le n$$

The above defined function f provides $(k, 1)$-mean labeling of the graph.

So, the quadrilateral snake Q_n is a $(k, 1)$-mean graph for all k.

Illustration 2.10

$(5, 1)$-mean labeling of Q_4 and $(6, 1)$-mean labeling of Q_5 are shown in Figure 2.8 and Figure 2.9 respectively.

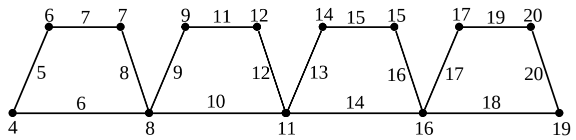

Figure 2.8: $(5, 1)$-ML of Q_4

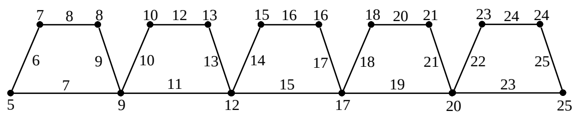

Figure 2.9: $(6, 1)$-ML of Q_5

Theorem 2.11

The graph $S(Q_n)$ obtained by the subdivision of each edge of the quadrilateral snake Q_n is a $(k, 1)$-mean graph for all k.

Proof

Let $V(S(Q_n)) = \{v_i,\ 1 \le i \le 2n,\ v_i^{'},\ 1 \le i \le 2n,\ u_i,\ 1 \le i \le n+1,\ w_i,\ 1 \le i \le n,\ w_i^{'},\ 1 \le i \le n\}$ be the vertices and $E(S(Q_n)) = \{a_i,\ 1 \le i \le 2n,\ b_i,\ 1 \le i \le 2n,\ c_i,\ 1 \le i \le 2n,\ d_i,\ 1 \le i \le 2n\}$ be the edges which are denoted as in Figure 2.10.

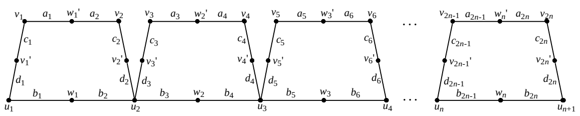

Figure 2.10: Ordinary labeling of $S(Q_n)$

First we label the vertices as follows:

Define $f : V \rightarrow \{0, 1, 2, ..., k + (q - 1)\}$ by

For $1 \le i \le 2n$,

$$f(v_i) = \begin{cases} k+4i, & i \text{ odd} \\ k+4i-3, & i \text{ even} \end{cases}$$

For $1 \le i \le n$

$$f\left(w_i^{'}\right) = k + 8i - 6$$

For $1 \le i \le 2n$,

$$f(v_i^{'}) = \begin{cases} k+4(i-1), & i \text{ odd} \\ k+4i-2, & i \text{ even} \end{cases}$$

For $1 \le i \le n + 1$,

$$f(u_i) = k + 8i - 9$$

For $1 \le i \le n$,

$$f(w_i) = k + 8i - 5$$

Then the induced edge labels are:

For $1 \le i \le 2n$,

$$f^*(a_i) = \begin{cases} k+4i-1, & i \text{ odd} \\ k+4(i-1), & i \text{ even} \end{cases}$$

$$f^*(c_i) = k + 4i - 2$$

$$f^*(d_i) = \begin{cases} k+4(i-1), & i \text{ odd} \\ k+4i-1, & i \text{ even} \end{cases}$$

$$f^*(b_i) = k + 4i - 3$$

The above defined function f provides $(k, 1)$-mean labeling of the graph.

So, the graph $S(Q_n)$ is a $(k, 1)$-mean graph for all k.

Illustration 2.12

$(10, 1)$-mean labeling of $S(Q_3)$ and $(11, 1)$-mean labeling of $S(Q_4)$ are shown in Figure 2.11 and Figure 2.12 respectively.

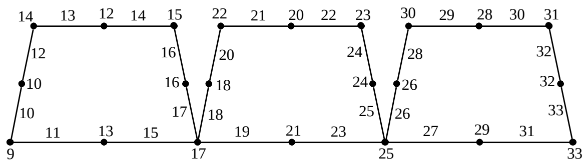

Figure 2.11: $(10, 1)$-ML of $S(Q_3)$

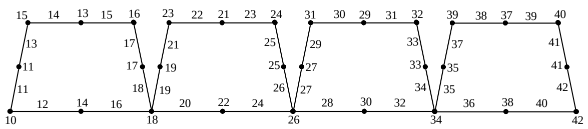

Figure 2.12: $(11, 1)$-ML of $S(Q_4)$

Definition 2.13 [28]

An alternate quadrilateral snake $A(Q_n)$ is obtained from a path $u_1, v_1, u_2, v_2, ..., u_n, v_n$ by joining u_i, v_i to new vertices $w_i, w_i^{'}$ respectively and then joining w_i and $w_i^{'}$. That is every alternate edge of a path is replaced by a cycle C_4.

Theorem 2.14

The alternate quadrilateral snake $A(Q_n)$ $(n \geq 2)$ is a $(k, 1)$-mean graph for all k.

Proof

Let $V(A(Q_n)) = \{u_i, v_i, w_i, w_i^{'}, 1 \leq i \leq n\}$ be the vertices and $E(A(Q_n)) = \{e_i, 1 \leq i \leq 2n-1, a_i, 1 \leq i \leq 2n, c_i, 1 \leq i \leq n\}$ be the edges which are denoted as in Figure 2.13.

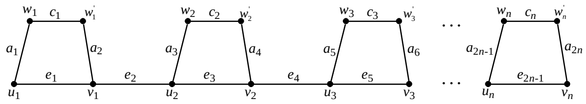

Figure 2.13: Ordinary labeling of $A(Q_n)$

First we label the vertices as follows:
Define $f : V \rightarrow \{0, 1, 2, ..., k + (q-1)\}$ by
For $1 \leq i \leq n$,

$$f(u_i) = k + 5(i-1);$$
$$f(v_i) = k + 5i - 2$$
$$f(w_i) = k + 5(i-1) - 1;$$
$$f\left(w_i^{'}\right) = k + 5i - 3$$

Then the induced edge labels are:
For $1 \leq i \leq 2n-1$,

$$f^*(e_i) = \begin{cases} k + 2 + \dfrac{5(i-1)}{2}, & i \text{ odd} \\ k + \dfrac{5i}{2} - 1, & i \text{ even} \end{cases}$$

For $1 \leq i \leq 2n$,

$$f^*(a_i) = \begin{cases} k + \dfrac{5(i-1)}{2}, & i \text{ odd} \\ k + \dfrac{5i}{2} - 2, & i \text{ even} \end{cases}$$

For $1 \leq i \leq n$,

$$f^*(c_i) = k + 5i - 4$$

The above defined function f provides $(k, 1)$-mean labeling of the graph.

So, the alternate quadrilateral snake $A(Q_n)$ $(n \geq 2)$ is a $(k, 1)$-mean graph for all k.

Illustration 2.15

$(5, 1)$-mean labeling of the alternate quadrilateral snake $A(Q_4)$ and $(10, 1)$-mean labeling of the alternate quadrilateral snake $A(Q_5)$ are shown in Figure 2.14 and Figure 2.15 respectively.

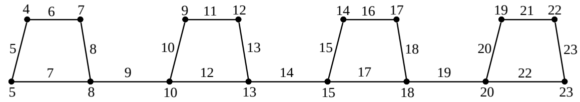

Figure 2.14: $(5, 1)$-ML of $A(Q_4)$

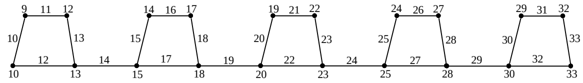

Figure 2.15: $(10, 1)$-ML of $A(Q_5)$

Theorem 2.16

The alternate double quadrilateral snake $A(D(Q_n))$ is a $(k, 1)$-mean graph for all k.

Proof

Let $V(A(D(Q_n))) = \{v_i, 1 \leq i \leq 2n, v_i^{'}, 1 \leq i \leq 2n, v_i^{''}, 1 \leq i \leq 2n\}$ be the vertices and $E(A(D(Q_n))) = \{e_i, 1 \leq i \leq 2n-1, a_i, b_i, 1 \leq i \leq 2n, c_i, d_i, 1 \leq i \leq n\}$ be the edges which are denoted as in Figure 2.16.

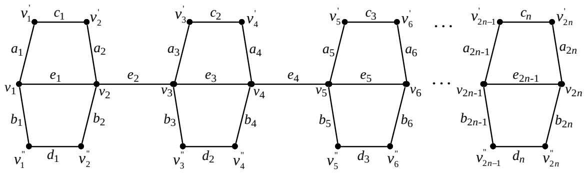

Figure 2.16: Ordinary labeling of $A(D(Q_n))$

First we label the vertices as follows:
Define $f : V \rightarrow \{0, 1, 2, ..., k + (q-1)\}$ by
For $1 \leq i \leq 2n$,

$$f(v_i) = \begin{cases} k + 4i - 2, & i \text{ odd} \\ k + 4i - 5, & i \text{ even} \end{cases}$$

$$f\left(v_i^{'}\right) = \begin{cases} k + 4i - 5, & i \text{ odd} \\ k + 4(i-2), & i \text{ even} \end{cases}$$

$$f\left(v_i^{''}\right) = \begin{cases} k + 4i + 1, & i \text{ odd} \\ k + 4i - 2, & i \text{ even} \end{cases}$$

Then the induced edge labels are:

$$f^*(e_i) = k + 4i - 1, \quad 1 \leq i \leq 2n-1$$

For $1 \leq i \leq 2n$,

$$f^*(a_i) = \begin{cases} k + 4i - 3, & i \text{ odd} \\ k + 4i - 6, & i \text{ even} \end{cases}$$

$$f^*(b_i) = \begin{cases} k + 4i, & i \text{ odd} \\ k + 4i - 3, & i \text{ even} \end{cases}$$

For $1 \leq i \leq n$,

$$f^*(c_i) = k + 8(i-1)$$
$$f^*(d_i) = k + 8i - 2$$

The above defined function f provides $(k, 1)$-mean labeling of the graph.

So, the alternate double quadrilateral snake $A(D(Q_n))$ is a $(k, 1)$-mean graph for all k.

Illustration 2.17

$(6, 1)$-mean labeling of alternate double quadrilateral snake $A(D(Q_3))$ and $(9, 1)$-mean labeling of alternate double quadrilateral snake $A(D(Q_4)$ are shown in Figure 2.17 and Figure 2.18 respectively.

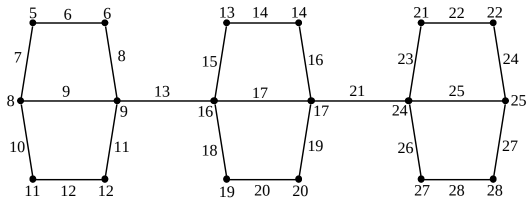

Figure 2.17: (6, 1)-ML of $A(D(Q_3))$

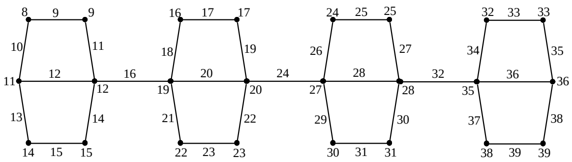

Figure 2.18: (9, 1)-ML of $A(D(Q_4))$

CONCLUSION

In this paper we investigated (k, 1)-mean labeling of some snake related graphs. All the results in this paper are novel. For the better understanding of the proofs of the theorems, labeling pattern defined in each theorem is demonstrated by illustration.

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